

HOMOGENEOUS ANR -SPACES AND ALEXANDROFF MANIFOLDS

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ABSTRACT. We specify a result of Yokoi [18] by proving that if G is an abelian group and X is a homogeneous metric ANR compactum with $\dim_G X = n$ and $\check{H}^n(X; G) \neq 0$, then X is an (n, G) -bubble. This implies that any such space X has the following properties: $\check{H}^{n-1}(A; G) \neq 0$ for every closed separator A of X , and X is an Alexandroff manifold with respect to the class D_G^{n-2} of all spaces of dimension $\dim_G \leq n-2$. We also prove that if X is a homogeneous metric continuum with $\check{H}^n(X; G) \neq 0$, then $\check{H}^{n-1}(C; G) \neq 0$ for any partition C of X such that $\dim_G C \leq n-1$. The last provides a partial answer to a question of Kallipoliti and Papasoglu [8].

1. INTRODUCTION

In this paper we establish some properties of homogeneous metric compacta. One of the main problems concerning homogeneous compacta is the Bing-Borsuk [2] question whether any closed separator of an n -dimensional homogeneous metric ANR -space is cyclic in dimension $n-1$. Yokoi's result [18, Theorem 3.3] provides a partial answer to this question. Our first result is a clarification of [18, Theorem 3.3], we omit the requirement G to be a principal ideal domain.

Theorem 1.1. *Let X be a homogeneous metric ANR -continuum with cohomological dimension $\dim_G X = n$ and $\check{H}^n(X; G) \neq 0$, where G is an abelian group. Then X is an (n, G) -bubble.*

Following Yokoi [18], a compactum X is called an (n, G) -bubble if $\check{H}^n(X; G) \neq 0$ and $\check{H}^n(A; G) = 0$ for every closed proper set $A \subset X$. This is a reformulation of the notion of an n -bubble introduced by Kuperberg [13] and Choi [4], see also Karimov-Repovš [9] for the stronger notion of an \check{H}^n -bubble.

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Corollary 1.2. *Let X be a homogeneous metric ANR-continuum with $\check{H}^n(X; G) \neq 0$ and $\dim_G X = n$. Then*

- (i) *X is a strong V_G^n -continuum;*
- (ii) *X is an Alexandroff manifold with respect to the class D_G^{n-2} of all spaces of dimension $\dim_G \leq n - 2$;*
- (iii) *If A is a closed separator of X , then $\check{H}^{n-1}(A; G) \neq 0$.*

Item (ii) from Corollary 1.2 was proved in [11] under the additional requirement that G is a principal ideal domain. Here, $\check{H}^n(X; G)$ denotes the reduced n -th Čech cohomology group of X with coefficients from G . We say that a set $A \subset X$ is massive if A has a non-empty interior in X .

Recall that a space X is an *Alexandroff manifold with respect to a class \mathcal{C}* , see [10] and [16], if for every two disjoint, closed massive sets $X_0, X_1 \subset X$ there exists an open cover ω of X such that there is no partition P in X between X_0 and X_1 admitting an ω -map onto a space $Y \in \mathcal{C}$. This definition is inspired by the Alexandroff's notion of V^n -continua [1] which is obtained when \mathcal{C} is the class of all compacta whose covering dimension is $\leq n - 2$.

A compactum X is said to be a V_G^n -continuum [15] if for every two open, disjoint subsets U_1, U_2 of X there exists an open cover ω of $X_0 = X \setminus (U_1 \cup U_2)$ such that any partition P in X between U_1 and U_2 does not admit an ω -map g of P onto a space Y with $g^*: \check{H}^{n-1}(Y; G) \rightarrow \check{H}^{n-1}(P; G)$ being trivial. If, in addition, there exists also an element $e \in \check{H}^{n-1}(X_0; G)$ such that for any partition P between U_1 and U_2 and any ω -map g of P onto a space Y we have $0 \neq i_P^*(e) \in g^*(\check{H}^{n-1}(Y; G))$, where i_P is the embedding $P \hookrightarrow X_0$, X is called a *strong V_G^n -continuum*. For example, every (n, G) -bubble is a strong V_G^n -continuum, see [11].

It follows directly from the above definitions that V_G^n -continua are Alexandroff manifolds with respect to the class D_G^{n-2} of all spaces of dimension $\dim_G \leq n - 2$. The converse is not true, for example the Menger n -dimensional compactum is a V^n -continuum, but it is not a V_G^n -continuum for any group G , see [11].

Homogeneous metric compacta (not necessary ANR) are also interesting class of spaces. Krupski [12] has shown that any such an n -dimensional space is a Cantor n -manifold. One of the ingredients of Krupski' proof is the classical result established by Hurewicz-Menger [6] and Tumarkin [17] that any n -dimensional compactum contains an n -dimensional Cantor n -manifold. Kuz'minov [14] provided a cohomological counterpart of this fact about V^n -continua (see [10] for more general results). Concerning V_G^n -continua we have the following statement which provides a positive answer of Question 4.3 from [11]:

Theorem 1.3. *Any compactum X with $\check{H}^n(X; G) \neq 0$ contains a strong V_G^n -continuum.*

Theorem 1.3 could be also compared with the cohomological version of Kuperberg's result [13, Theorem 5.5] that any n -dimensional n -cyclic metric compactum X for which $\check{H}^n(X; \mathbb{Z})$ is finitely generated (in particular, any n -dimensional n -cyclic ANR) contains an n -bubble.

The condition $\check{H}^n(X; G) \neq 0$ in the above theorem is essential. For example, let X be the square \mathbb{I}^2 and suppose X contains a (strong) $V_{\mathbb{Z}}^2$ -continuum K , where \mathbb{Z} is the group of all integers. Then $\dim K = 2$, so K contains a non-empty interior U in X . Now, take a segment \mathbb{I} joining two opposite sides of \mathbb{I}^2 and intersecting U . Obviously $\mathbb{I} \cap K$ is a partition of K . Since $\check{H}^{n-1}(P; G) \neq 0$ for every partition P of a V_G^n -continuum, $\check{H}^1(\mathbb{I} \cap K; \mathbb{Z}) \neq 0$. On the other hand, because $\mathbb{I} \cap K$ is an one-dimensional subset of \mathbb{I} , $\check{H}^1(\mathbb{I} \cap K; \mathbb{Z}) = 0$.

Kuperberg [13] asked whether any n -dimensional metric compactum contains an $(n-1)$ -bubble. This question is still open, but the following corollary provides a result in this direction.

Corollary 1.4. *Any compactum X with $\dim_G X = n$ contains a strong V_G^{n-1} -continuum.*

For finite-dimensional metric compacta and V_G^n -continua Theorem 1.3 and Corollary 1.4 were established in [15].

Proposition 1.5. *Let X be a homogeneous metric continuum with $\check{H}^n(X; G) \neq 0$. Then for any partition C of X there exists an open cover ω of X such that C does not admit any ω -map $g: C \rightarrow Y$ onto a space of dimension $\dim_G Y \leq n-1$ with $g^*: \check{H}^{n-1}(Y; G) \rightarrow \check{H}^{n-1}(C; G)$ being a trivial homomorphism.*

Let us note that the n -dimensional universal Menger compactum μ^n , which is a homogeneous continuum cyclic in dimension n , contains a separator C such that $\dim C = n$ and $\check{H}^{n-1}(C; G) = 0$, see [11, Corollary 2.6]. Therefore, the restriction $\dim_G Y \leq n-1$ in Proposition 1.5 and the condition $\dim_G P \leq n-1$ in next corollary are essential.

Corollary 1.6. *Let X be a homogeneous metric continuum such that $\check{H}^n(X; G) \neq 0$. Then $\check{H}^{n-1}(P; G) \neq 0$ for every partition P of X with $\dim_G P \leq n-1$.*

Kallipoliti and Papasoglu [8] have shown that every 2-dimensional locally connected, simply connected homogeneous metric continuum can not be separated by an arc, and asked if the simple connectedness can be dropped from this result. Corollary 1.6 provides a partial answer to the Kallipoliti-Papasoglu question.

2. COHOMOLOGICAL CARRIERS

In this section we consider cohomological carriers of non-trivial elements of $\check{H}^n(X; G)$ and establish some properties of them. We fix an abelian group G , an integer n and a metric compactum X with $\check{H}^n(X; G) \neq 0$. A closed non-empty set $A \subset X$ is said to be a *cohomological carrier* (shortly, a carrier) of a non-zero element $\alpha \in \check{H}^n(X; G)$ if $i_A^*(\alpha) \neq 0$ and $i_B^*(\alpha) = 0$ for every proper closed subset $B \subset A$, where i_A denotes the inclusion map $A \hookrightarrow X$.

Lemma 2.1. *For every non-zero element $\alpha \in \check{H}^n(X; G)$ there exists a carrier. Moreover, if A is a carrier of α , then $\check{H}^{n-1}(P; G) \neq 0$ for any closed partition P of A .*

Proof. The first part of Lemma 2.1 follows from Zorn's lemma and the continuity of Čech cohomology. For the second part, suppose A is a carrier of α and P a partition of A . Then there exist two closed proper subsets A_1 and A_2 of A such that $A = A_1 \cup A_2$ and $P = A_1 \cap A_2$. Consider the Mayer-Vietoris exact sequence

$$\check{H}^{n-1}(P; G) \rightarrow \check{H}^n(A; G) \rightarrow \check{H}^n(A_1; G) \oplus \check{H}^n(A_2; G).$$

For every $i = 1, 2$ let $\partial_i: \check{H}^n(A; G) \rightarrow \check{H}^n(A_i; G)$ be generated by the inclusion $A_i \hookrightarrow A$. Denote also by Δ and φ , respectively, the left and right homomorphism of the above sequence. Since each A_i is a proper subset of A we have $\varphi(\beta) = (\partial_1(\beta), \partial_2(\beta)) = 0$, where $\beta = i_A^*(\alpha)$. So, there exists $\gamma \in \check{H}^{n-1}(P; G)$ with $\Delta(\gamma) = \beta$. Because β is a non-trivial element of $\check{H}^n(A; G)$, so is γ . Hence, $\check{H}^{n-1}(P; G) \neq 0$. \square

Everywhere below, if $B \subset A$, then $i_{A,B}$ denotes the inclusion $B \hookrightarrow A$. The next lemma is an analogue of Lemma 4 from [4].

Lemma 2.2. *Let $A \subset X$ be a carrier of a non-trivial element $\alpha \in \check{H}^n(X; G)$ and B a closed subset of X . Then $A \subset B$ if and only if $\text{Ker}(j_B^*) \subset \text{Ker}(j_A^*)$, where $j_A = i_{A \cup B, A}$ and $j_B = i_{A \cup B, B}$ are the corresponding inclusions.*

Proof. Obviously $A \subset B$ implies $\text{Ker}(j_B^*) \subset \text{Ker}(j_A^*)$. Suppose that $\text{Ker}(j_B^*) \subset \text{Ker}(j_A^*)$, but B does not contain A . Then $A \cap B$ is a proper closed subset of A (possibly empty). The left homomorphism in the Mayer-Vietoris exact sequence

$$\check{H}^n(A \cup B; G) \rightarrow \check{H}^n(A; G) \oplus \check{H}^n(B; G) \rightarrow \check{H}^n(A \cap B; G).$$

is defined by (j_A^*, j_B^*) , while the right one i^* assigns to each $(\beta_1, \beta_2) \in$

$\check{H}^n(A; G) \oplus \check{H}^n(B; G)$ the difference $i_{A, A \cap B}^*(\beta_1) - i_{B, A \cap B}^*(\beta_2)$. Since $A \cap B$ is a proper subset of A , $i_{A \cap B}^*(\alpha) = 0$. Then $i^*((i_A^*(\alpha), 0)) = 0$. Consequently, there exists $\beta \in \check{H}^n(A \cup B; G)$ with $(j_A^*(\beta), j_B^*(\beta)) = (i_A^*(\alpha), 0)$. So, $\beta \in \text{Ker}(j_B^*)$ and, according to our assumption, $\beta \in \text{Ker}(j_A^*)$. The last relation contradicts $i_A^*(\alpha) \neq 0$. Therefore, $A \subset B$. \square

The next proposition is actually Theorem 5 from [4]. We provide a different proof of that theorem.

Proposition 2.3. *Let $A \subset X$ be a carrier for a non-trivial element of $\check{H}^n(X; G)$ and $f: X \rightarrow X$ a map homotopic to the identity on X . If $\dim_G X \leq n$, then $A \subset fA$.*

Proof. By [7], we can identify the cohomological group $\check{H}^n(A \cup fA; G)$ with the group $[A \cup fA, K(G, n)]$ of homotopy classes from $A \cup fA$ to $K(G, n)$, where $n > 0$ and $K(G, n)$ denotes an Eilenberg-MacLane complex. Similarly, $\check{H}^n(A; G)$ and $\check{H}^n(fA; G)$ are identified with the groups $[A, K(G, n)]$ and $[fA, K(G, n)]$, respectively.

By Lemma 2.2, it suffices to prove that if $\alpha \in \text{Ker}(j_{fA}^*)$ then $\alpha \in \text{Ker}(j_A^*)$. So, we fix $\alpha \in \check{H}^n(A \cup fA; G)$ with $j_{fA}^*(\alpha) = 0$. According to the above identifications, there exists a map $g: A \cup fA \rightarrow K(G, n)$ such that $\alpha = [g]$. Since $\dim_G X \leq n$, g can be extended to a map $\tilde{g}: X \rightarrow K(G, n)$. Because $j_{fA}^*(\alpha) = 0$, we can find a homotopy $H_1: fA \times [0, 1] \rightarrow K(G, n)$ with $H_1(x, 0) = g(x)$ and $H_1(x, 1) = *$ for all $x \in fA$, where $*$ is a point from $K(G, n)$. Then the homotopy $H_2: A \times [0, 1] \rightarrow K(G, n)$, $H_2(x, t) = H_1(f(x), t)$, connects the constant map $\kappa: A \hookrightarrow *$ and the map $h: A \rightarrow K(G, n)$ defined by $h(x) = g(f(x))$. Next, consider a homotopy $F: A \times [0, 1] \rightarrow X$ with $F(x, 0) = f(x)$ and $F(x, 1) = x$, and define $H: A \times [0, 1] \rightarrow K(G, n)$ by $H(x, t) = \tilde{g}(F(x, t))$. We have $H(x, 0) = g(f(x))$ and $H(x, 1) = g(x)$ for all $x \in A$. Hence, H is connecting the maps h and the restriction $g|_A$ of g over A . Finally, combining H and H_2 , we can produce a homotopy on A connecting the map $g|_A$ and the constant map κ . Hence, $j_A^*(\alpha) = [g|_A] = 0$. \square

Before proving the next property of carriers, we introduce some more notations. If ω is a finite open cover of a closed set $Z \subset X$, we denote by $|\omega|$ and p_ω , respectively, the nerve of ω and a map from Z onto $|\omega|$ generated by a partition of unity subordinated to ω . Furthermore, if $C \subset Z$ and $\omega(C) = \{W \cap C : W \in \omega\}$, then $p_{\omega(C)}: C \rightarrow |\omega(C)|$ is the restriction $p_\omega|_C$. Recall also that p_ω generates maps $p_\omega^*: \check{H}^k(|\omega|; G) \rightarrow \check{H}^k(Z; G)$ for $k \geq 0$. Moreover, if $q_\omega: Z \rightarrow |\omega|$ is a map generating

by (another) partition of unity subordinated to ω , then p_ω and q_ω are homotopic. So, $p_\omega^* = q_\omega^*$.

Proposition 2.4. *Let K be a carrier for a non-trivial element of $\alpha \in \check{H}^n(X; G)$. Then for any two open disjoint subsets U_1 and U_2 of K there exists an open cover ω of $K \setminus (U_1 \cap U_2)$ and an element $\eta \in \check{H}^{n-1}(|\omega|; G)$ such that $p_{\omega(C)}^*(i_{\omega(C)}^*(\eta)) \neq 0$ for every partition C of K between U_1 and U_2 , where $i_{\omega(C)}$ is the inclusion $|\omega(C)| \hookrightarrow |\omega|$.*

Proof. Let $K_1 = K \setminus U_1$, $K_2 = K \setminus U_2$, C be a partition of K between U_1 and U_2 , and F_1, F_2 closed subsets of K such that: $F_1 \cap F_2 = C$, $F_1 \cup F_2 = K$, $F_1 \subset K_1$ and $F_2 \subset K_2$. Consider the commutative diagram whose rows are Mayer-Vietoris sequences:

$$\begin{array}{ccccc} \check{H}^{n-1}(K_1 \cap K_2; G) & \xrightarrow{\delta} & \check{H}^n(K; G) & \xrightarrow{j} & \check{H}^n(K_1; G) \oplus \check{H}^n(K_2; G) \\ \downarrow i^* & & \downarrow id & & \downarrow i_1^* \oplus i_2^* \\ \check{H}^{n-1}(C; G) & \xrightarrow{\delta_1} & \check{H}^n(K; G) & \xrightarrow{j_1} & \check{H}^n(F_1; G) \oplus \check{H}^n(F_2; G). \end{array}$$

Since $j(\beta) = 0$, where $\beta = i_K^*(\alpha)$, there exists a non-zero element $\gamma \in \check{H}^{n-1}(K_1 \cap K_2; G)$ with $\delta(\gamma) = \beta$. Consequently, we can find an open cover ω of $K_1 \cap K_2$ and a non-trivial element $\eta \in \check{H}^{n-1}(|\omega|; G)$ such that $p_\omega^*(\eta) = \gamma$. It follows from the commutativity of the above diagram that $i^*(\gamma) \neq 0$. Then the equality $p_{\omega(C)}^*(i_{\omega(C)}^*(\eta)) = i^*(\gamma)$ completes the proof. \square

Proposition 2.5. *Every carrier for a non-trivial element of $\check{H}^n(X; G)$ is a strong V_G^n -continuum.*

Proof. Indeed, suppose U_1 and U_2 are open subsets of K having disjoint closures. Let ω be an open cover of $K_0 = K \setminus (U_1 \cup U_2)$ and $e \in \check{H}^{n-1}(|\omega|; G)$ a non-trivial element satisfying the hypotheses of Proposition 2.4. Assume C a partition of K between U_1 and U_2 admitting an ω -map g onto a space T . Thus, we can find a finite open cover τ of T such that $\nu = g^{-1}(\tau)$ is refining ω . Let $p_\nu: C \rightarrow |\nu|$ be a map onto the nerve of ν generated by a partition of unity subordinated to ν . Obviously, the function $V \in \tau \rightarrow g^{-1}(V) \in \nu$ provides a simplicial homeomorphism $g_\nu^\tau: |\tau| \rightarrow |\nu|$. Then the maps p_ν and $g_\tau = g_\nu^\tau \circ \pi_\tau \circ g$, where $\pi_\tau: T \rightarrow |\tau|$ is a map generated by a partition of unity subordinated to $|\tau|$, are homotopic. Hence, $p_\nu^* = g^* \circ \pi_\tau^* \circ (g_\nu^\tau)^*$.

On the other hand, since ν refines ω , we can find a map $\varphi_\nu: |\nu| \rightarrow |\omega(C)|$ such that $p_{\omega(C)}$ and $\varphi_\nu \circ p_\nu$ are homotopic. Therefore, $p_{\omega(C)}^* = p_\nu^* \circ \varphi_\nu^*$. According to Proposition 2.4, there exists $\eta \in \check{H}^{n-1}(|\omega|; G)$

with $p_{\omega(C)}^*(i_{\omega(C)}^*(\eta)) \neq 0$. Since $i_C^*(p_\omega^*(\eta)) = p_{\omega(C)}^*(i_{\omega(C)}^*(\eta))$, $e = p_\omega^*(\eta)$ is a non-zero element of $\check{H}^{n-1}(K_0; G)$. Here $p_\omega: K_0 \rightarrow |\omega|$ is a map generated by a partition of unity subordinated to ω and $i_C: C \hookrightarrow K_0$ is the inclusion map. Moreover, both equalities $p_\nu^* = g^* \circ \pi_\tau^* \circ (g_\nu^\tau)^*$ and $p_{\omega(C)}^* = p_\nu^* \circ \varphi_\nu^*$ yield that $i_C^*(e)$ is a non-trivial element of $g^*(\check{H}^{n-1}(T; G))$. \square

3. PROOF OF THEOREM 1.1 AND COROLLARY 1.2

Proof of Theorem 1.1. Suppose G is an abelian group, X is a non-trivial homogeneous metric ANR-continuum with $\dim_G X = n$ and $\check{H}^n(X; G) \neq 0$. Since X is an ANR, $n \geq 1$ and there exists a positive ϵ such that any two ϵ -close maps from X into X are homotopic (we say that two maps $f_1, f_2: X \rightarrow X$ are ϵ -close if $\text{dist}(f_1(x), f_2(x)) < \epsilon$ for each $x \in X$).

It suffices to show that if A is a carrier for a non-trivial element $\alpha \in \check{H}^n(X; G)$, then $A = X$. Indeed, suppose there exists a proper subset $B \subset X$ with $\check{H}^n(B; G) \neq 0$, and choose a non-trivial element $\beta \in \check{H}^n(B; G)$. Since $\dim_G X = n$, there exists $\alpha \in \check{H}^n(X; G)$ with $\beta = i_B(\alpha)$. Because the carrier of α is X and B is a proper subset of X , $i_B(\alpha) = 0$, a contradiction.

Next, suppose $A \subset X$ is a carrier for a non-trivial $\alpha \in \check{H}^n(X; G)$ and A is a proper set. According to the Effros' theorem [5], there corresponds a positive number δ with the following property: whenever x and y are points from X and $\text{dist}(x, y) < \delta$, there is a homeomorphism $h: X \rightarrow X$ such that $h(x) = y$ and h is ϵ -close to the identity id_X on X . Because $A \neq X$, we can choose points $a \in A$ and $b \notin A$ with $\text{dist}(a, b) < \delta$. Consequently, there would be a homeomorphism $f: X \rightarrow X$ such that $f(a) = b$ and $\text{dist}(f, \text{id}_X) < \epsilon$. Obviously, $g = f^{-1}: (X, fA) \rightarrow (X, A)$ generates the isomorphism $g^*: \check{H}^n(X; G) \rightarrow \check{H}^n(X; G)$ and $B = f(A)$ is a carrier for the element $g^*(\alpha)$. Moreover, the homeomorphism g is also ϵ -close to id_X . Hence, g is homotopic to id_X . Applying Proposition 2.3 to the carrier B and the homeomorphism g , we obtain that $B \subset g(B) = A$ which contradicts $b \in B \setminus A$. Hence, A should be the whole space X . \square

Proof of Corollary 1.2. Item (i) follows from Proposition 2.5. Item (ii) follows from the simple observation that any V_G^n -continuum is an Alexandroff manifold with respect to the class D_G^{n-2} . Since X is a carrier for every non-trivial $\alpha \in \check{H}^n(X; G)$, item (iii) follows from Lemma 2.1. \square

4. V_G^n -CONTINUA

In this section we provide the proofs of Theorem 1.3, Corollary 1.4 and Propositions 1.5-1.7.

Proof of Theorem 1.3. Since $\check{H}^n(X; G) \neq 0$, X contains a carrier A of a non-trivial element of $\check{H}^n(X; G) \neq 0$. Then, according to Propositions 2.5, A is a strong V_G^n -continuum. \square

Proof of Corollary 1.4. This corollary follows directly from Theorem 1.3 because every compactum X with $\dim_G X = n$ contains a closed subset F such that $\check{H}^{n-1}(F; G) \neq 0$ (see, for example, [14]). \square

Proof of Proposition 1.5. Suppose there exists a partition C of X such that for every open cover ω of X , C admits an ω -map $g_\omega: C \rightarrow Y_\omega$ onto a space of dimension $\dim_G Y_\omega \leq n - 1$ with $g_\omega^*: \check{H}^{n-1}(Y_\omega; G) \rightarrow \check{H}^{n-1}(C; G)$ being a trivial homomorphism. Then, according to [3, Theorem 2.4], $\dim_G C \leq n - 1$. Obviously, the boundary B of C in X is also a partition of X and $\dim_G B \leq n - 1$. Moreover, we have the commutative diagram below, where $g_\omega|B: B \rightarrow g_\omega(B)$ is the restriction of g_ω

$$\begin{array}{ccc} \check{H}^{n-1}(Y_\omega; G) & \xrightarrow{g_\omega^*} & \check{H}^{n-1}(C; G) \\ \downarrow i_{g(B)}^* & & \downarrow i_B^* \\ \check{H}^{n-1}(g_\omega(B); G) & \xrightarrow{(g_\omega|B)^*} & \check{H}^{n-1}(B; G) \end{array}$$

Since $\dim_G Y_\omega \leq n - 1$, $i_{g(B)}^*$ is a surjection. This implies that $(g_\omega|B)^*$ is the trivial homomorphism because so is g_ω^* . Therefore, considering B instead of C , we may assume that C does not have interior points in X . The above diagram also shows that for every closed subset $A \subset C$ and every ω the restriction $g_\omega|A$ is an ω -map onto $g_\omega(A)$ such that $(g_\omega|A)^*: \check{H}^{n-1}(g_\omega(A); G) \rightarrow \check{H}^{n-1}(A; G)$ is the trivial homomorphism.

By Theorem 1.3, there exists a strong V_G^n -continuum $K \subset X$. Since X is homogeneous, we may also assume that $K \cap C \neq \emptyset$. Observe that $z \in K \setminus C$ for some z . Indeed, the inclusion $K \subset C$ would imply that if P is a partition of K and γ any open cover of K , then P admits a γ -map h_γ onto a space T such that $(h_\gamma)^*: \check{H}^{n-1}(T_\gamma; G) \rightarrow \check{H}^{n-1}(P; G)$ is trivial. This would contradict the fact that K is a strong V_G^n -continuum. Let $X \setminus C = U \cup V$ and $z \in V$, where U and V are nonempty, open and disjoint sets in X . Then the Effros theorem [5] allows us to push K towards U by a small homeomorphism $h: X \rightarrow X$ so that the image $h(K)$ meets both U and V (see the proof of Lemma 2 from [12] for a similar application of Effros' theorem). Therefore, $S = h(K) \cap C$ is a partition of $h(K)$ such that for any ω the restriction $g_\omega|S$ is

an ω -map generating a trivial homomorphism $(g_\omega|S)^*$, a contradiction.

□

Proof of Corollary 1.6. It follows directly from Proposition 1.5. □

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